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Synchronization and chaos in the systems of two inductively connected thermo-resistor auto-generators

Beregov R.Y., Melkikh A.V.
Ural Federal University, Yekaterinburg, Russia,

Abstract. - We consider a system of inductively coupled oscillators on the basis of self-heating. It is demonstrated that the system has a metastable chaos. A metastable phase state that has a spectrum of Lyapunov exponents (0.0123, 0.0006, -0.2019, -0.223, -3.9748, -8.2018) and a fractional Lyapunov dimension ($D_L = 0.98$) is found. A strange non-chaotic attractor, which has a fractal structure (the correlation dimension is $D_C = 1.24$) and zero dominant Lyapunov exponents, is also found. The presence of two limit cycles is revealed. It is shown that synchronization is characteristic for the identical semiconductors and relatively weak force of inductive coupling.

Keywords: *- auto-generator,Lyapunov numbers, strange non-chaotic attractor,synchronization*

I. INTRODUCTION

S-shaped current-voltage characteristics exist in semiconductor systems, due to theself-heating ofthe samplestudiedrepeatedly(see,eg.[1-9]).In particular,in [7], it was shown that in such a system auto-oscillations may exist. However, synchronization, de-synchronizationand chaos inensembles of suchauto-generators are practically unexplored. As is known,in the case wherethere are multipleinteractingoscillatorsin the system, such phenomena as synchronization (see,eg.[10-15]), de-synchronization, as well as variouschaotic regimes (see,eg. [16-19]), take place. In addition, theevolution of such asystem over timesignificantly dependson the type ofconnectionbetween auto-generators. Inthis paper, we studya system of twoinductivelyinteractingauto-generators basedon the mechanism ofself-heating.

II. A SYSTEM OF TWO INDUCTIVELY COUPLED OSCILLATORS BASED ON THE MECHANISM OF SELF-HEATING

Consider a system consistingof twoidenticalinductive couplingsemiconductors, represented by sufficiently thin bars (suchthat the area ofthe endsis muchsmaller than that ofthe other faces).The point modelfor onesemiconductorgenerator [7] contains threeequations for threedimensionless variables(sample temperature, voltage and current):

$$\frac{dT}{dt} = IU - T + 1$$

$$z\frac{dI}{dt} = U - I\exp\left(\frac{E_{g0}}{T}\right) \qquad (1)$$

$$y\frac{dU}{dt} = I_{in} - I$$

where $C\dfrac{\alpha 2HL_s}{cm\sigma_0} = y, \dfrac{\alpha 2H\sigma_0 L_s L}{cm} = z$,- are dimensionless constants that, respectively, representthe ratio ofthe characteristic times ofthe processes of accumulation of charge(capacitivetime)and the inductance to the time ofheat exchange; H, L_s are the width and length of the bar.

Consider a situation wheretwo samplesmayinteractinductively; thus, we can write the equation

$$I_1 = \sigma_0\left(U_1 - L_1\frac{dI_1}{dt} - M_{12}^*\frac{dI_2}{dt}\right)\exp\left(-\frac{E_{g01}}{2kT_1}\right) \quad (2)$$

Accordingly,equations of the systemwill take the form:

$$
\begin{aligned}
&\frac{dT_1}{dt} = I_1U_1 - T_1 + 1 && \frac{dT_2}{dt} = I_2U_2 - T_2 + 1\\
&z_1\frac{dI_1}{dt} = U_1 - I_1\exp\left(\frac{E_{g01}}{T_1}\right) - M_{12}\frac{dI_2}{dt} && z_2\frac{dI_2}{dt} = U_2 - I_2\exp\left(\frac{E_{g02}}{T_2}\right) - M_{12}\frac{dI_1}{dt}\\
&y_1\frac{dU_1}{dt} = I_{in1} - I_1 && y_2\frac{dU_2}{dt} = I_{in2} - I_2
\end{aligned}
\tag{3}
$$

where $M_{12} = \dfrac{\alpha 2H\sigma_0 L_s M_{12}^*}{cm}$, $C\dfrac{\alpha 2HL_s}{cm\sigma_0} = y$, $\dfrac{\alpha 2H\sigma_0 L_s L_1}{cm} = z_1$, $\dfrac{\alpha 2H\sigma_0 L_s L_2}{cm} = z_2$

III. DYNAMICAL MODEL OT THE SYSTEM

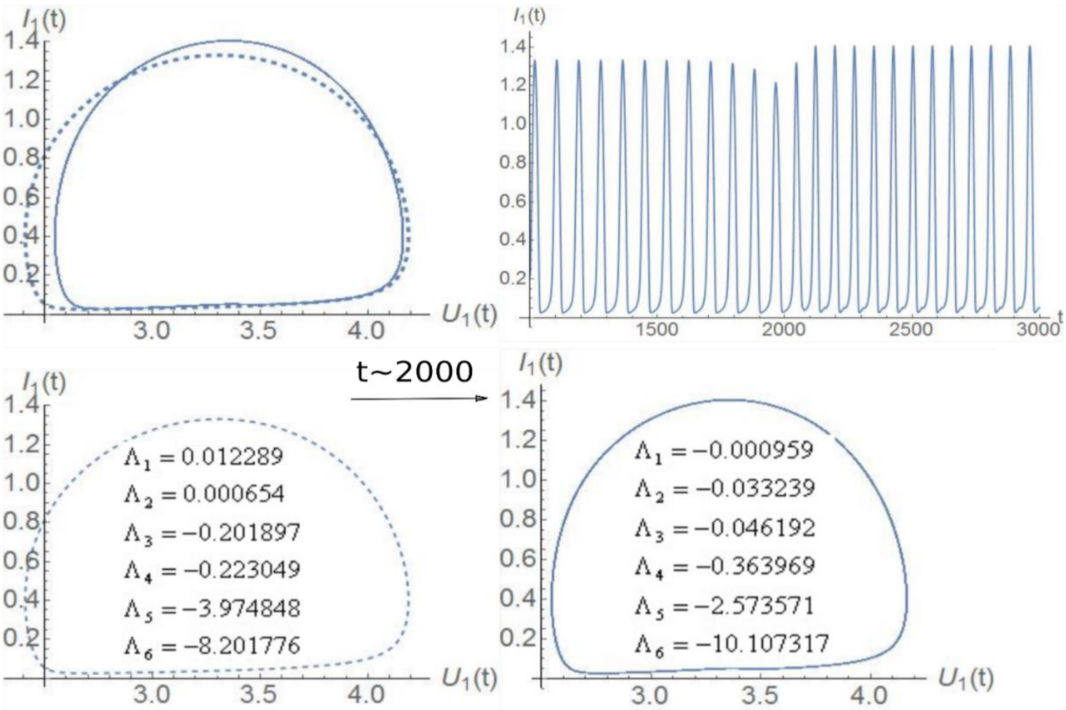
Numericalsolution (LSODE- Livermore Solver forOrdinary Differential Equations [20]approach with accuracy in 10^{-8}, control of numerical results is performed in Mathematica and MathLab programs) of the system (3) shows that the system of inductivelycoupled semiconductorauto-generatorshasa sufficientpool of complexattracting sets. Let us illustrate thiswith a few examplesofthe qualitative behavior ofthe systemof two coupledauto-generators.

First,it was found that, if the systemconsisting ofidenticaloscillatorsis inidentical conditionsandfor a sufficientlystrongattractiveforceconnection, it first enters ametastable phase curve, which,after stayingon it for some period oftime,"dumped" (presumably due to the limitedaccuracyof the computer simulation) in theattractor of thelimiting cycle type. As a result, chaosin the systemdisappears.

Let us illustrate thiseffectin severalareaswithexamplesof parametric dependencies (Table 1).

Table 1.Parameters and constants of the example.

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	5	10	10	0.4	0.4	4



IC: $T_1(0) = T_2(0) = 1$, $I_1(0) = I_2(0) = -1$, $U_1(0) = U_2(0) = -1$

Fig. 1.Illustrationof the system beingon a metastable phase curve with asubsequent transition to alimiting cycle.

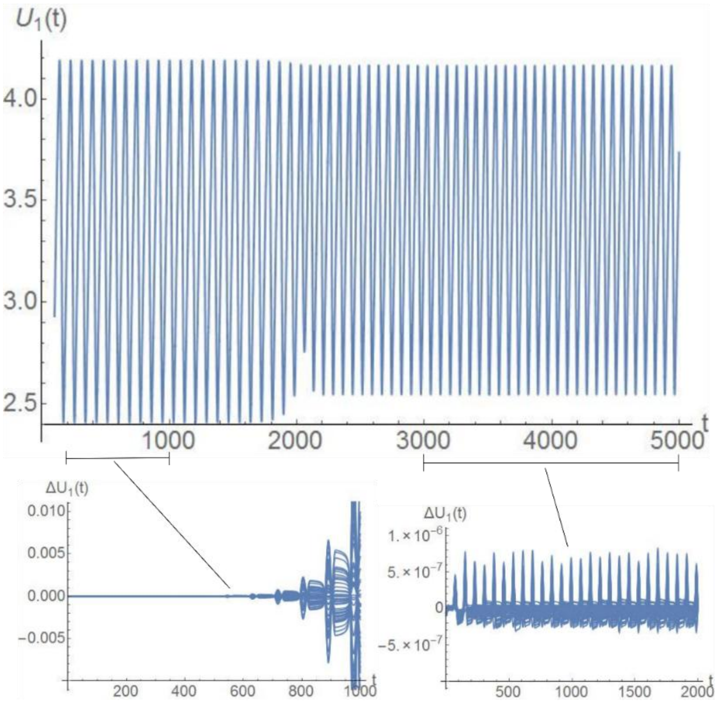


Fig. 2.Illustration ofinstability with respectto a small changein the initial conditionsat time points of the system stayon a metastable phase curveandafter the transition tothe limit cycle. The graphs(multiple modeling) show the bias of the potential differenceby varying theinitial conditionswithin$\epsilon \in (-10^{-8}, 10^{-8})$. The graph clearly shows the growth of the deviation of fluctuationsof the potential differencein thetimeintervalt $\in$ (0,2000)and sustainedoscillationswith a very smallconstant deviationforthe rest ofthe range.

Calculation ofKolmogorov-Sinai entropy also points tochaos(entropy is positive ($h \cong 0.012572$)) at the initial stage,with a subsequent transitionto the normalperiodic motion(entropy is zero ($h \cong 0.000089$)).

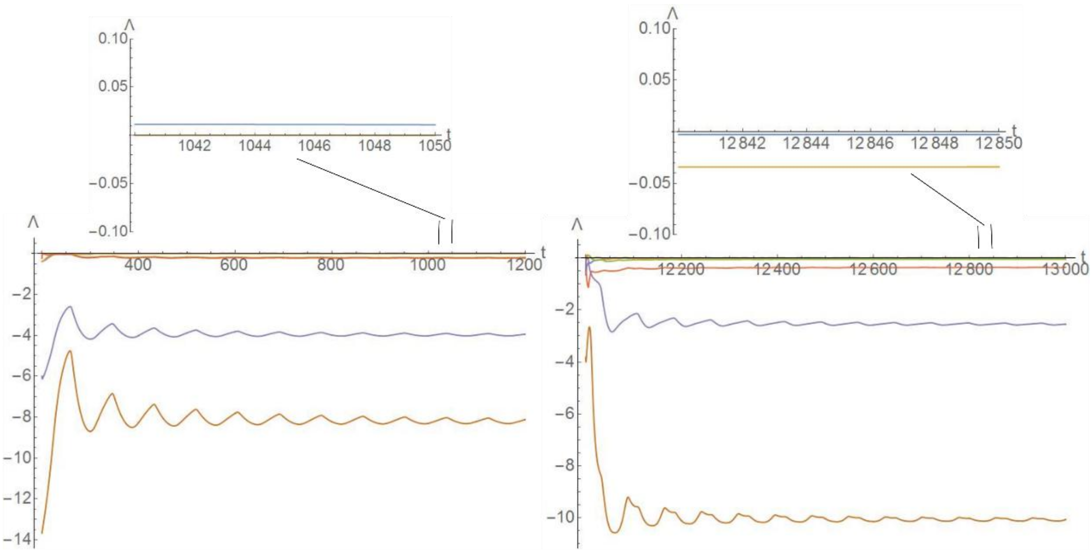


Fig. 3.Graphs of convergenceof Lyapunov numbers indifferent time intervals.

The standard deviationof the Lyapunov numbers [21, 22] was calculated on the base of fivevalues.

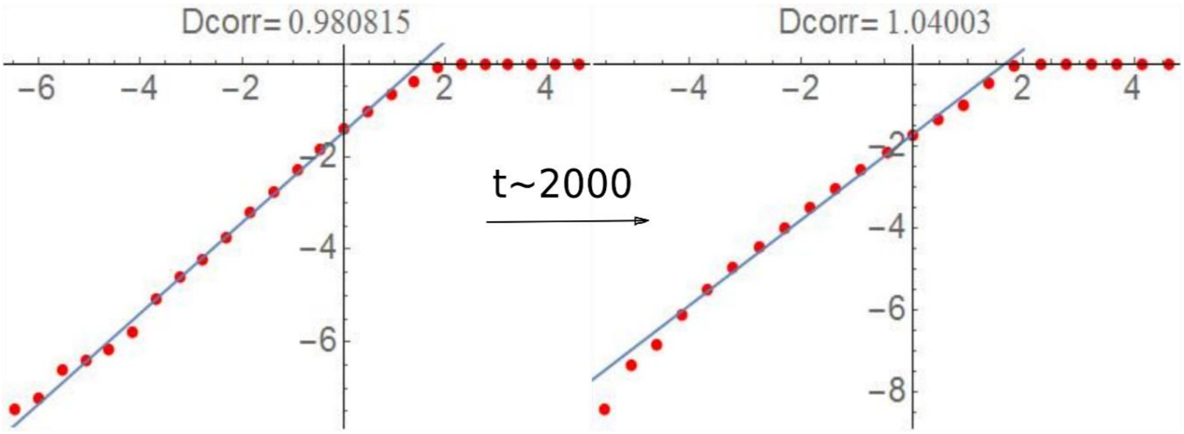


Fig. 4Correlation dimensionof attractors.

It should be notedthat thecorrelation dimension of the non-chaotic attractor, to which the systemfalls, is an integer value. Thus, theattractorin whichthe systemwould besome time later(t ≈2000)has an integerdimensionand a zerodominantLyapunov number; thus, this isa limiting cycle.
Secondly, the numerical simulation showedthat the poolof attractionhas a complex structure (Table 2).

Table 2.Parameters of the example (Fig. 2).

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	5	10	10	0.4	0.4	1

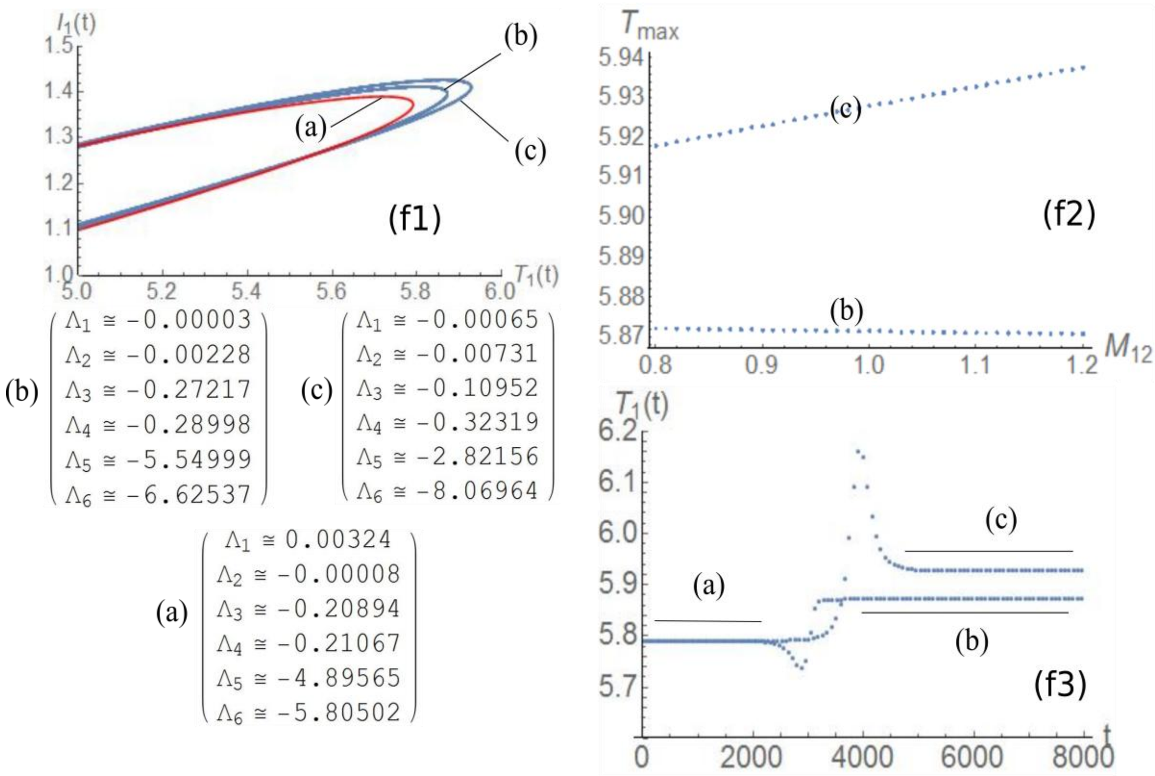


Fig. 5.This exampleshows the presence ofcomplexbasins of attraction. The upper right part of drawing (f2) is the bifurcation chart calculated for a local maximum of temperature depending on the force of inductive interaction. The leftpart of the figure(f1, f3)indicates a veryclosecoexistence ofchaoticmetastable phase curve andnon-chaotic attractors(b, c).

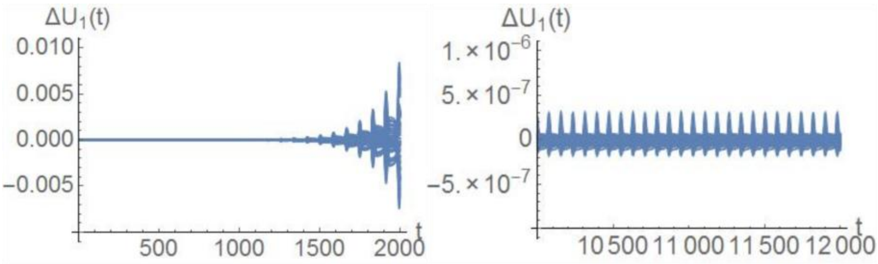


Fig. 6.Illustration of instability in relation to a small change in the initial conditions in moments of time being on a metastable phase curve and after the transition to the limiting cycle. The graphs show the potential difference deviation by varying the initial conditions within$\epsilon \in (-10^{-8}, 10^{-8})$.

These examples areindicative ofa complex topology of basinsof attraction. A part of thephase curvesare chaotic, as evidenced by the spectrum ofLyapunovnumbersof the form (+, 0, -, -,-, -)and instabilityto small variationsin the initial conditions. Due to the limitedaccuracyof numerical calculations ofthe systemfor a certain periodof timeis alwayson theattractors, wherethe largest Lyapunov exponentis zero.Both examplesindicate the presence ofunstablechaos.

Calculate thearea ofparametric dependenceof the existenceof unstablechaosfor several conditions (Table 3).

Table 3.Parameters of the example (Fig. 7).

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	5	10	10	0.4	0.4	$M \in (-9.5, 9.5)$

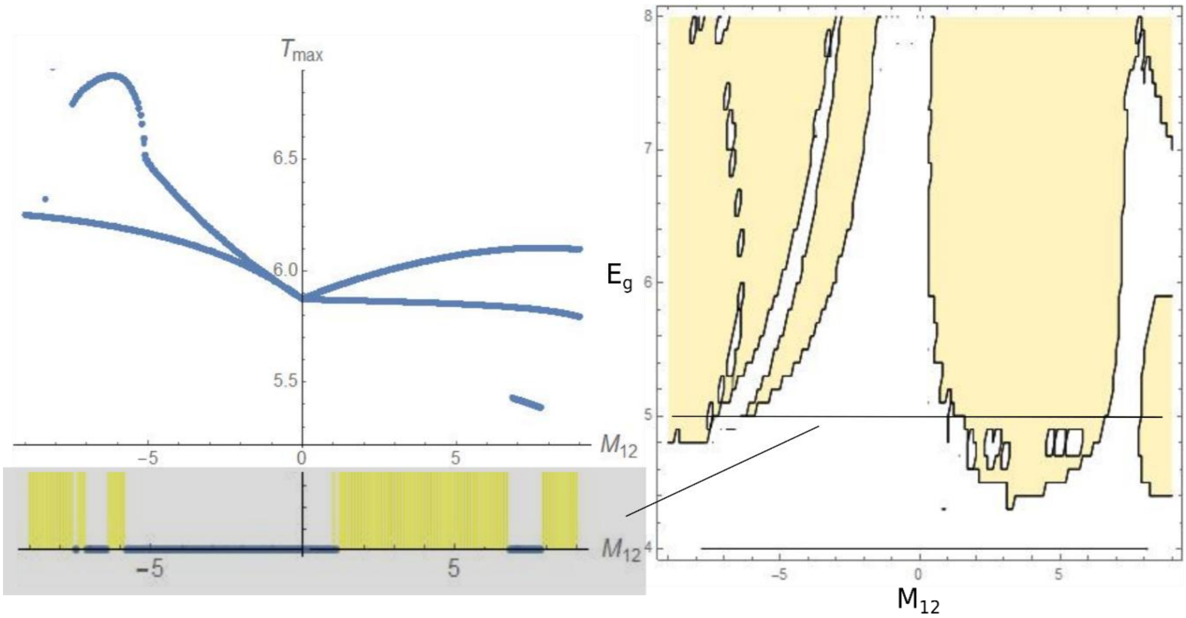


Fig. 7.Areas of metastablechaos(see also [17]) depending on thebandgapandinductive couplingstrength.

The basin of attraction ofthe systemis complicated,aschaotic metastable phase curvecoexistwithlimiting cycles(Fig. 1, 5).The spectrum ofLyapunovnumbersof the form (+, 0, -, -,-, -)and the instabilityto small variations(Fig.2, 6) indicate the presence of non-hyperbolicchaoticattractors. Timespent by the systemon suchmetastable phase curveis limited and,under certain conditions,quite considerable($T \approx 2000$, presumably because of the limitedaccuracy of numericalcalculations), hencethe chaos is unstable.However, it is impossibletocall such long-running processes transitional because the graphic of behavior (Fig. 1, 5) of the system in the transition fromchaotic metastable phase curvetolimiting cyclesshowsmuch less time($T \approx 1000$) than that of being on the chaotic metastable phase curve. This leads to the need to considersuch phenomena andstudy them.

IV. CHAOS IN THE SYSTEM OF COUPLING AUTO-GENERATORS

Along with theunstable chaos, stable chaos was foundfor a system consistingofnotidenticalsemiconductors. For example, thechaotic attractorcan be obtainedwith the following parametersand initial conditions (Table 4).

Table 4.Parametersand constants.

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	6.1	10	10	0.4	0.4	5

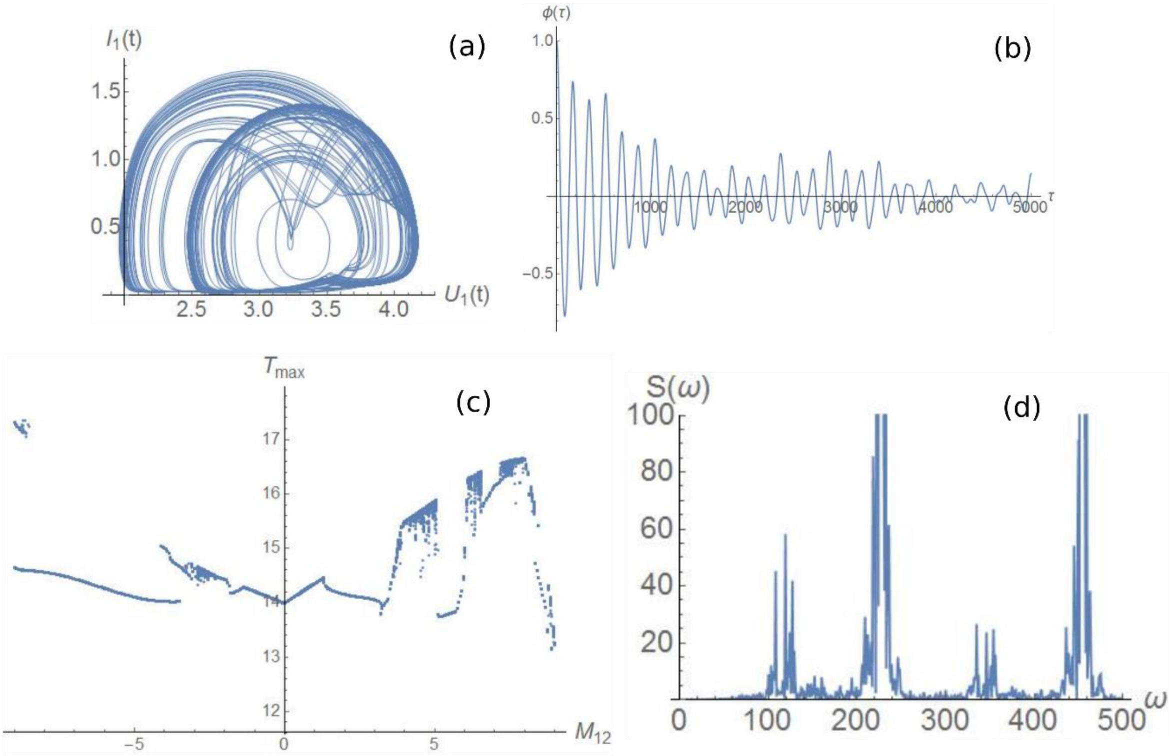


Fig. 8.Chaotic attractor(a),bifurcation diagram(c),the power spectrum(d), the autocorrelation function(b).

Fig. 8shows a typicalnon-hyperbolicchaoticattractor because theLyapunov number spectrum is as follows:(+,0, -, -, -, -, -); there is a splitting of thecorrelations (Fig. 8b) andthe power spectrum(Fig. 8d).

The presence of a chaoticstableattractor was found for the system of inductivelyinteractingsemiconductors(Fig.8), and it has the following properties: fractionalLyapunov dimension[22] $D_L \approx 2.019$, splittingcorrelations (autocorrelation functions of (Fig.8b)show adeclinefrom the envelope, but it obviouslydoes not tend tozero) and, continuous (the profileis very uneven) power spectrum(Fig.8 d.), correspond toa non-hyperbolicsystem type(+,0, -, -, -, -).

V. SYNCHRONIZATION OF AUTO-OSCILLATIONS

Let us calculate the area ofsynchronizationbased onlongtransition timestosimple limitingcycles.We carry outresearchto identify theparametric dependenceof different typesof synchronization for the fixed parameters ofTab.5.

Table 5.Fixed parameters(current and constant).

z	y	I_{in1}	I_{in2}
10	10	0.4	0.4

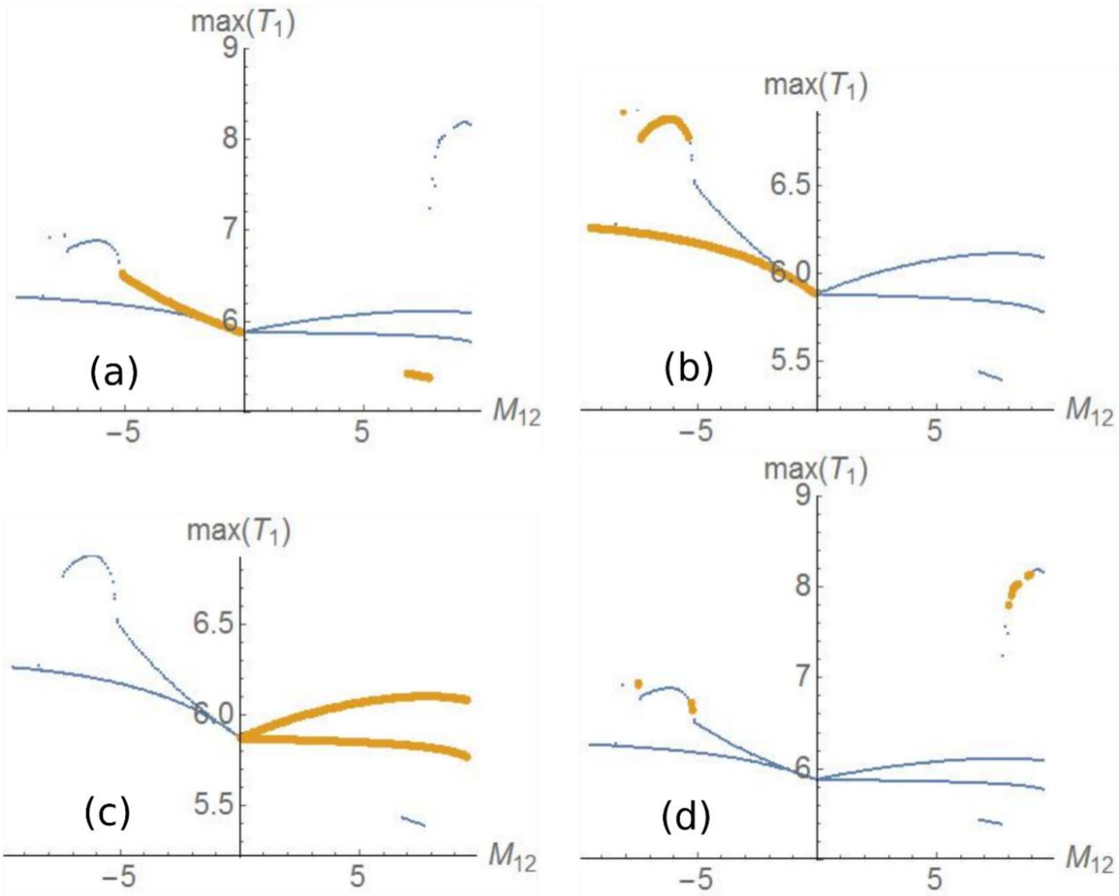


Fig. 9. Areas of various types of synchronization and de-synchronization ((a)-full in-phase synchronization, (b)-full sync in antiphase, (c)-synchronization, (d)-desync), depending onthe coupling strength (the band gap $E_{g1} = E_{g2} = 5$).

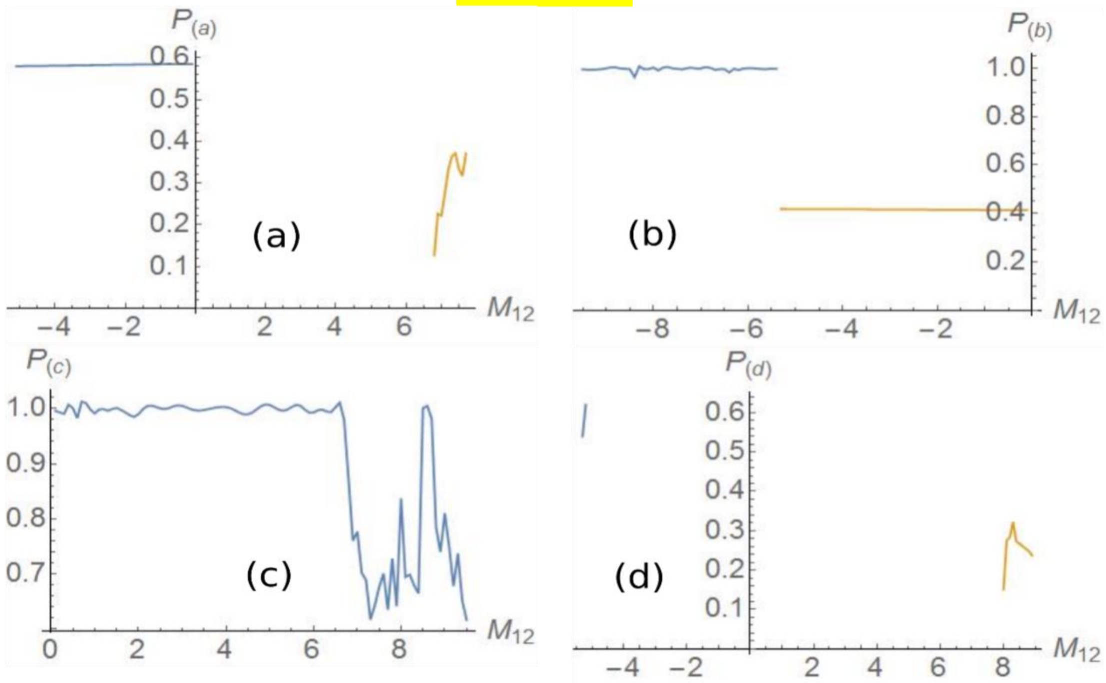


Fig. 10. The probability of various types of synchronization and de-synchronization ((a)-full in-phase synchronization, (b)-full sync in antiphase, (c)-synchronization, (d)-desync), depending onthe coupling strength (the band gap $E_{g1} = E_{g2} = 5$).

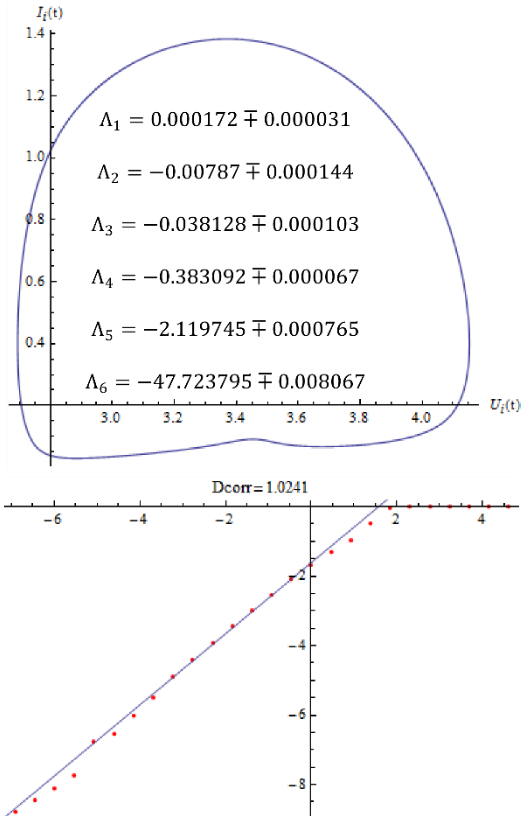
In the case of identical auto-generators (Fig. 9), there are different types of synchronization, namely the in-phase, antiphase and effect similar to the lag-synchronization (we also see a complete in-phase synchronization with delay on the phase of oscillations) [23]. For the realization of the in-phase synchronization and in-phase synchronization with delay, small forces and the relatively weak dependence of the band-gap are needed, whereas antiphase synchronization is implemented with a strong attracting connection.

VI. STUDY OF THE BASIN OF ATTRACTION OF NON-CHAOTIC ATTRACTING SETS

The systemrevealed the presence ofseveraltypes ofnon-chaoticattractors of limiting cycle-type,which can be obtained, for example, under the following conditions (Table 6).

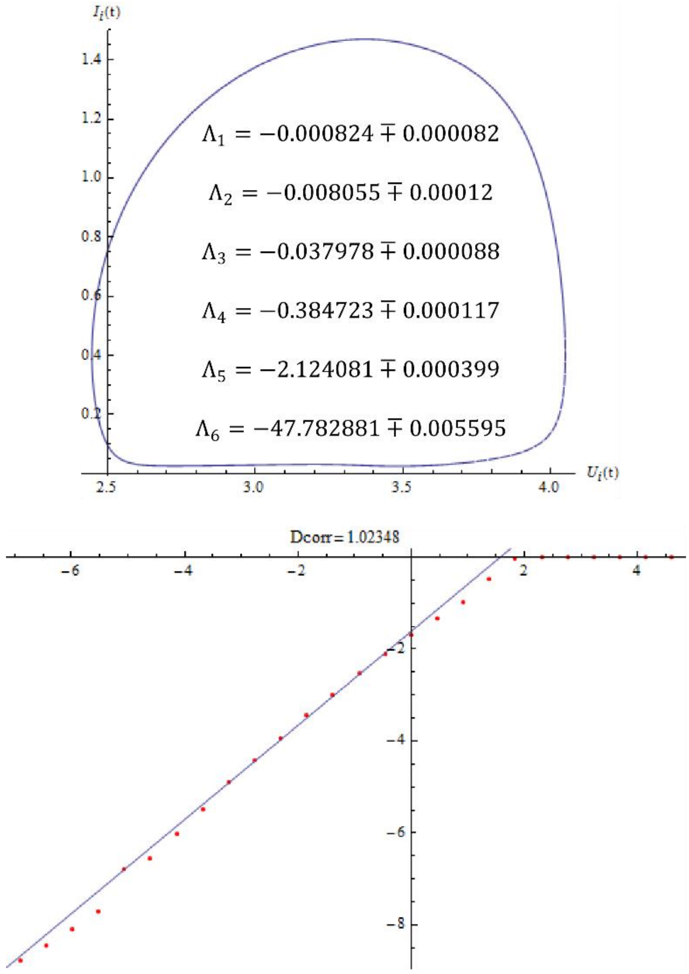
Table6.Systems parameters

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	5	10	10	0.4	0.4	9



IC: $T_1(0) = 1, I_1(0) = 1, U_1(0) = 1 \quad T_2(0) = 1, I_2(0) = -1, U_2(0) = 1$

Fig. 11.Limiting cycle, zerodominantLyapunov exponentand integercorrelation dimensionof the attractor.



IC: $T_1(0) = 1, I_1(0) = 1, U_1(0) = 1 \quad T_2(0) = 1, I_2(0) = -1, U_2(0) = -1$

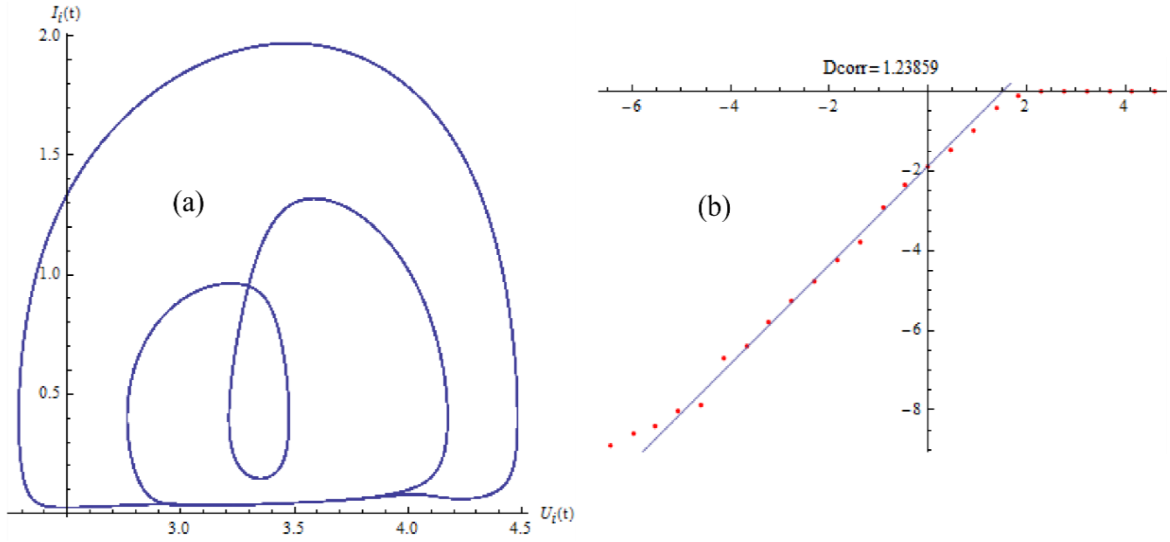
Fig. 12.Limiting cycle, zerodominantLyapunov exponentand the wholecorrelation dimensionof the attractor.

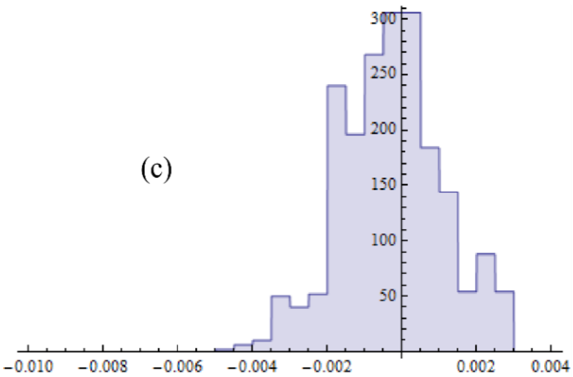
VII. STRANGE NON-CHAOTIC ATTRACTOR

Among attractorsofthe system,a strangenon-chaoticattractor(SNA, see, for example, [24, 25]) was discovered for the case of the identical semiconductors. This type ofattractoris characterizedmainly by a fractional dimensionandnon-positivedominant(averaged over the entire attractor, while the distribution ofthe local exponent canhave a 'tail' inthe positive half-axis) Lyapunov exponent. To receive thisattractor is possible by setting the following parameters(Table7) andthe initial conditions.

Table7.Parameters of the system of auto-generators

E_{g1}	E_{g2}	z	y	I_{in1}	I_{in2}	M_{12}
5	5	10	10	0.4	0.4	9





$$\Lambda_1 = 0.00023 \mp 0.000031$$

$$\Lambda_2 = -0.00827 \mp 0.000144$$

$$\Lambda_3 = -0.037845 \mp 0.000103$$

$$\Lambda_4 = -0.382905 \mp 0.000067$$

$$\Lambda_5 = -2.117544 \mp 0.000765$$

$$\Lambda_6 = -47.702096 \mp 0.008067$$

IC: $T_1(0) = 1, I_1(0) = 1, U_1(0) = 1 \quad T_2(0) = 1, I_2(0) = 10, U_2(0) = 1$

Fig. 13.The strangenon-chaoticattractor of the system, (a) – is the attractor, (b) – is the correlation integral, (c) – is the distribution ofthe dominantlocalLyapunov numbers.

It should be notedthat the system consistsofidenticalband-gapsemiconductors;however,thereexists anSNA-typeattractor.

VIII. CONCLUSIONS

Thus, the system of inductively coupled auto-generators demonstrates metastable chaos (Fig. 5), the residence time on which is estimated as $T \approx 2000$ dimensionless units (Fig. 1). In addition to the metastable chaos, a chaotic attractor is found (Fig. 8), which has a spectrum of Lyapunov exponents (0.0044, -0.0005, -0.1535, -0.4971, -3.9358, -33.6911) and a fractional Lyapunov dimension ($D_L = 2.026$). A strange non-chaotic attractor is found (Fig. 13, for identical auto-generators), which has a fractal structure (the correlation dimension is equal to $D_C = 1.24$) and a zero dominant Lyapunov exponent ($\Lambda = 0.00023$). The presence of two limiting cycles (Fig. 11, 12, the dominant exponents are, respectively, equal to $\Lambda = 0.00017$, $\Lambda = -0.00082$ and the correlation dimensions are $D_C = 1.024$, $D_C = 1.023$) is revealed, i.e., the system is bistable. The onset of synchronization is typical for the same characteristic of semiconductors and the relatively weak force of inductive coupling ($|M_{12}| < 5$ Fig. 9).

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